60/64

AUS | الجامعة الأمريكية في الشارقة
American University of Sharjah

Department of Mathematics & Statistics

December 18, 2019

Linear Algebra (Math221) | Fall 2019
Time: 2 hours | Final Exam

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INSTRUCTIONS: Please read and follow the following instructions. A violation of any of these instructions may lead to questioning your academic integrity.

- Do not open this exam until you are instructed to begin.
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- This exam has 6 pages PLUS this cover page.
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Circle the name of your professor and the time of your class				
Al-Sharawi	Badawi	Kallel	Louhichi	Uygul

Total Score:________ /64

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Final Exam , MTH 221 , Fall 2019

Ayman Badawi

SCORE = $\frac{\quad}{64}$

(a.-2ae$^{\pi i}$ba) **(4 points)** Let $A = \begin{bmatrix} 1 & 1 & 7 \\ -2 & (2+r) & 8 \\ -1 & -1 & r \end{bmatrix}$. If possible find all values of r, such that the system $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} r^2 \\ r+7 \\ 3r \end{bmatrix}$ has unique solution. If such r does not exist, then explain briefly. $\rightarrow |A| \neq 0$

$A \xrightarrow[R_1+R_3 \to R_3]{2R_1+R_2\to R_2} \begin{bmatrix} 1 & 1 & 7 \\ 0 & (4+r) & 22 \\ 0 & 0 & (7+r) \end{bmatrix}$ $\boxed{B}$

$|B| = |A|$

$|B| = 1 * (4+r)(7+r)$

$|A| = 28 + 11r + r^2$

for unique solution $\rightarrow |A| \neq 0$

$28 + 11r + r^2 \neq 0$

$\boxed{r \neq -7 \text{ and } r \neq -4}$

* r can be any real number except $r \neq -7$ & $r \neq -4$

(b.-2be$^{\pi i}$bb) **(6 points)** Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 2 & -1 & 2 \\ 0 & 2 & -3 \end{bmatrix}$. If possible find A^{-1}. If A^{-1} exists, then find $(A^T)^{-1}$. $(A^T)^{-1} = (A^{-1})^T$

$[A_{n\times n} | I_n]$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & -2 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 & 1 & 0 \\ 0 & 2 & -3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{R_1+R_2\to R_2} \left[\begin{array}{ccc|ccc} 0 & 1 & -2 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 0 & 1 & -2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{2R_3+R_1\to R_1}$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -3 & 0 & 2 \\ 1 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{\text{rearrange rows}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 1 & 0 & -3 & 0 & 2 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ -3 & 0 & 2 \\ -2 & 0 & 1 \end{bmatrix} \rightarrow (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} 1/2 & -3 & -2 \\ 1/2 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

(c.-$2ce^{\pi i}bc$) Given A is 4×5 such that $A \xrightarrow{-3R_2}$ $\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$ $\begin{bmatrix} 0 & 1 & 1 & 1 & 2 \\ 1 & -2 & -1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(aa) **(4 points)** Find the solution set to the homogeneous system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and write it as SPAN.

read

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & c \end{array}$$
$$\left[\begin{array}{ccccc|c} 0 & \boxed{1} & 1 & 1 & 2 & 0 \\ 1 & -2 & -1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{2R_1 + R_2 \to R_2} \left[\begin{array}{ccccc|c} 0 & \boxed{1} & 1 & 1 & 2 & 0 \\ \boxed{1} & 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = -x_3 - 4x_4 - x_5$

$x_2 = -x_3 - x_4 - 2x_5$

$x_3, x_4, x_5 \in \mathbb{R}$

Solution set $= \text{Span}\{(-1,-1,1,0,0), (-4,-1,0,1,0), (-1,-2,0,0,1)\}$

solution set $= \{(-x_3 - 4x_4 - x_5, -x_3 - x_4 - 2x_5, x_3, x_4, x_5) \mid x_3, x_4, x_5 \in \mathbb{R}\}$

(bb) **(3 points)** What is the independent number (dimension) of the solution set? What is the Rank(A)?

IN(Sol. set) = 3 Rank(A) = 2

$\mathbb{R}^4$

(d.-$2de^{\pi i}bd$) Let $D = \{A \in R^{2\times2} \mid A^T = -A\}$. $\longrightarrow \mathbb{R}^{2\times2}$

(bπ 1) **(4 points)** Convince me that D is a subspace of $R^{2\times2}$ by rewriting D as a span of some 2×2 matrices.

$D' = \{(a,b,c,d) \mid (a,c,b,d) = (-a,-b,-c,-d), a,b,c,d \in \mathbb{R}\}$

$D' = \{(a,b,c,d) \mid a = -a, b = -c, d = -d, a,b,c,d \in \mathbb{R}\}$

$D' = \{(-a,-c,c,-d) \mid a,c,d \in \mathbb{R}\}$ 3

$D' = \text{span}\{(-1,0,0,0), (0,-1,1,0), (0,0,0,-1)\}$

translate

$D = \text{span}\left\{\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\right\}$

(bπ 2) **(2 points)** Find IN(D) (i.e., $dim(D)$)

$\boxed{\text{IN(D)} = 3}$

(e.-2ee$^{\pi i}$be) Let $A = \begin{bmatrix} 4 & 4 & -8 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

(d1π) **(2 points)** Find all eigenvalues of A.

$$C_A(\alpha) = |\alpha I_n - A| = \left| \begin{bmatrix} \alpha-4 & -4 & 8 \\ 0 & \alpha-5 & 0 \\ 0 & 0 & \alpha-5 \end{bmatrix} \right|$$

$$C_A(\alpha) = (\alpha-4)(\alpha-5)(\alpha-5)$$

$\Rightarrow$ eigen values of A $\Rightarrow$ $C_A(\alpha)=0$ $\Rightarrow$ $\boxed{\begin{matrix} \alpha = 4 & \text{rep } 1 \\ \alpha = 5 & \text{rep } 2 \end{matrix}}$

(d2π) **(3 points)** For each eigenvalue a of A, find a basis for E_a (i.e, find a basis for the eigenspace that corresponds to the eigenvalue a)

$* E_4$ = solution set to $(\alpha I_n - A)Q_T = 0$ $\Rightarrow$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & C \end{array}$$
$$\left[\begin{array}{ccc|c} 0 & -4 & 8 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{4}R_1} \left[\begin{array}{ccc|c} 0 & 1 & -2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \xrightarrow{R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 0 & 1 & -2 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right] \xrightarrow[2R_2+R_1\to R_1]{R_2+R_3\to R_3} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_2 = 0$
$x_3 = 0$
$x_1 \in \mathbb{R}$

$E_4 = \{(x_1, 0, 0) \mid x_1 \in \mathbb{R}\}$ $\Rightarrow$ $\boxed{E_4 = \text{span}\{(1,0,0)\}}$

$* E_5$ = solution set of

$$\begin{array}{cccc} x_1 & x_2 & x_3 & C \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -4 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow x_1 = 4x_2 - 8x_3$$

$x_2, x_3 \in \mathbb{R}$

$E_5 = \{(4x_2 - 8x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R}\}$ $\Rightarrow$ $\boxed{E_5 = \text{span}\{(4,1,0), (-8,0,1)\}}$

Basis for $\Rightarrow$ E_4 is $\{(1,0,0)\}$ & E_5 is $\{(4,1,0), (-8,0,1)\}$

(d3π) **(2 points)** If A is diagnolizable, then find a diagonal matrix D and invertible (nonsingular) matrix Q such that $A = QDQ^{-1}$ (i.e., $D = Q^{-1}AQ$)

A is diagnolizable $\Rightarrow$ IN (E_α) = # α is repeated as root of $C_A(\alpha)=0$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 4 & -8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(f.-2fe$^{\pi i}$bf) **(3 points)** Let $A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & -4 & -1 \\ -1 & -2 & 0 \end{bmatrix}$. Find the LU-Factorization of A(i.e, find an upper triangular matrix U and lower triangular matrix L such that $A = LU$).

$$A \underset{R_1+R_3\to R_3}{\overset{R_1+R_2\to R_2}{\sim}} \begin{bmatrix} 1 & 2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \Leftarrow U \text{ matrix}$$

$$A \xrightarrow[R_1+R_3\to R_3]{R_1+R_2\to R_2} U \qquad \xleftarrow[-R_1+R_3\to R_3]{-R_1+R_2\to R_2}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underset{-R_1+R_3\to R_3}{\overset{-R_1+R_2\to R_2}{\sim}} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \Leftarrow L \text{ matrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(g.-2ge$^{\pi i}$bg) **(2 points)** Let A be 2×4 matrix such that $A \xrightarrow{3R_1} B \xrightarrow{-2R_2+R_1\to R_1} C$. Find two elementary matrices E_1, E_2 such that $E_1E_2A = C$.

$-2R_2+R_1\to R_1$ I_2 $\quad$ $3R_1$ I_2 $\quad$ 2×2 $\;2\times2$ $\;2\times4$

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} A = C$$

(h.-2he$^{\pi i}$bh) Consider the linear transformation $T : R^3 \to R^4$ such that $T(a,b,c) = (a+2c, -b+c, -a+b, -2a-4c)$. Then

(3aπ) **(1 point)** Find the Standard Matrix Presentation of T.

$$M_S = \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \\ -2 & 0 & -4 \end{bmatrix} \end{array}$$

(3bπ) **(2.5 points)** Write Range(T) as span of independent points and find $IN(Range(T))$

Range(T) = Col(M_S)

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \\ -2 & 0 & -4 \end{bmatrix} \underset{2R_1+R_4\to R_4}{\overset{R_1+R_3\to R_3}{\sim}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \overset{-1R_2}{\sim} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \overset{-R_2+R_3\to R_3}{\sim}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \overset{\frac{1}{3}R_3}{\sim} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \boxed{1} & 0 & 2 \\ 0 & \boxed{1} & -1 \\ 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

* Range(T) = span$\{(1,0,-1,-2), (0,-1,1,0), (2,1,0,-4)\}$

* IN(Range(T)) = 3

(3cπ) **(2 points)** Is T one-to-one (i.e., injective)? Is T onto (i.e., surjective)? Is T bijective (isomorphism)? Explain BRIEFLY.

IN(Range) < IN(codomain) $\Rightarrow$ not onto

IN(domain) = IN(Z(T)) + IN(Range(T))

3 $\quad$ 3

$\therefore$ IN(Z(T)) = 0 $\xRightarrow{\text{yes}}$ one to one

since not onto $\Rightarrow$ not bijective

$(a,b,c) \to R^3$

(i.-2ie$^{\pi i}$bi) Consider the linear Transformation $T : P_3 \to P_2$ such that $T(ax^2+bx+c) = (a-b-3c)x+(-2a+2b+6c)$

$\to (a-b-3c, -2a+2b+6c)$ R^2

($a^2\pi$) **(1 points)** Find the **FAKE** standard matrix presentation of T

translate

$T(a,b,c) = (a-b-3c, -2a+2b+6c)$

fake standard matrix presentation

$$M_s = \begin{array}{ccc} a & b & c \end{array} \begin{bmatrix} 1 & -1 & -3 \\ -2 & 2 & 6 \end{bmatrix}$$

($b^2\pi$) **(3 points)** Use Gram-Schmidt and find an orthogonal basis for $Z(T)(Ker(T))$, use the **fake**-dot product on P_3 (i.e., $(ax^2+bx+c).(dx^2+mx+n) = ad+bm+cn$)

$Z(T) =$ solution set to $M_s \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -3 & 0 \\ -2 & 2 & 6 & 0 \end{array}\right] \xrightarrow{2R_1+R_2 \to R}$

$$\begin{array}{ccc} a & b & c \end{array} \left[\begin{array}{ccc|c} 1 & -1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} a = b+3c \\ b, c \in \mathbb{R} \end{array}$$

$Z(T) = \{(b+3c, b, c) \mid b, c \in \mathbb{R}\} \Rightarrow Z(T) = \text{Span}\{(1,1,0),(3,0,1)\}$

Basis for $Z(T) = \{\overset{Q_1}{(1,1,0)}, \overset{Q_2}{(3,0,1)}\}$

orthogonal basis for $Z(T) = \{w_1, w_2\} = \{(1,1,0),(3,-3,2)\}$

translate $\Leftrightarrow$ $\boxed{\text{Orthogonal basis for } Z(T) = \{x^2+x, 3x^2-3x+2\}}$

Part II: SHORT ANSWER, STARE REALLY WELL and THINK, EACH 1.5 point, I will ONLY stare at the answer and not at your work

(i.ie^i) (This item is 2 points) Let $T : R^2 \to R^3$ be a linear transformation such that $T(1,3) = (1,0,-1)$ and $T(-1,1) = (-1,4,9)$. Then the starndard matrix presentation of T is ($\begin{bmatrix} 1 & 0 \\ -3 & 1 \\ -7 & 2 \end{bmatrix}$) and $T(2,-2)$ = ($(2,-8,-18)$)

$$\left[\begin{array}{cc|ccc} 1 & 3 & 1 & 0 & -1 \\ -1 & 1 & -1 & 4 & 9 \end{array}\right] \xrightarrow{R_1+R_2 \to R_2} \left[\begin{array}{cc|ccc} 1 & 3 & 1 & 0 & -1 \\ 0 & 4 & 0 & 4 & 8 \end{array}\right] \xrightarrow{\frac{1}{4}R_2} \left[\begin{array}{cc|ccc} 1 & 3 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 2 \end{array}\right]$$

(ii.iie$^i i$) Let A be a 2×2 matrix such that $A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$ and $A\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Then $|A|$ = (12).

(iii.iiie$^i ii$) If A is a 3×3 matrix and $|A| = -2$. Then $|-3A^2A^T|$ = (216).

(iv.ive$^i v$) **Write down T or F** . If $T : R^4 \to P_4$ is a linear transformation that is ONTO, then T is one-to-one (T)

(v.ve^v) If B is 2×2 and $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} B = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$. Then B = ($\begin{bmatrix} 3 & 4 \\ 0 & -2 \end{bmatrix}$)

2

Z^{-1} Z^{-1}

(vi.vie$^v i$) If A is a diagnolizable matrix 7×7 matrix and $C_A(\alpha) = (\alpha-3)^3(\alpha+1)^4$, then $IN(E_3)$ = (3) and $IN(E_{-1})$ = (4)

rep 3 rep 4

(vii.viie^vii) (This item is 2 points) If $B = \{x^3, x^3+x+1, f_1(x), f_2(x)\}$ is a basis for P_4, where $deg(f_1) = deg(f_2) = 3$, then a possibility for f_1 = (x^3+x^2) and a possibility for f_2 = (x^3+2)

2

viii.viiie^viii) Let $D = span\{\begin{bmatrix}1 & 2\\0 & -1\end{bmatrix}, \begin{bmatrix}-2 & -4\\0 & 2\end{bmatrix}, \begin{bmatrix}-1 & -2\\0 & 1\end{bmatrix}\}$. Then $IN(D)$ $(dim(D))$ = (1)

(ix.ixe^ix) (this item is 2 points) Given A is 3×3 and $C_A(\alpha) = (\alpha-3)^3$, where $(3,0,0) \in E_3$ (i.e., (3, 0, 0) is an eigenpoint of A) Let $F = I_3 + 6A^{-1} + A^2$. Then an eigenvalue of F is (12) and it corresponds to the eigenpoint (3, 0, 0) of F.

2

(x.xe^x) **Write down F OR T**. Suppose that $AB = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ for some 3×3 matrices A and B, then $|A| = 0$ AND $|B| = 0$ (F)

(xi.xie^xi) Let A be a 4×4 matrix such that $A \xrightarrow{2R_3+R_1\to R_1} \overset{B}{\begin{bmatrix}0 & 1 & 2 & 2\\4 & 0 & 5 & 2\\0 & -1 & 2 & 2\\0 & -1 & -2 & -4\end{bmatrix}}$. Then $|A|$ = (32)

$$\begin{array}{c}R_1+R_3\to R_3\\ \sim\\ R_1+R_4\to R_4\end{array} \underset{C}{\begin{bmatrix}0 & 1 & 2 & 2\\4 & 0 & 5 & 2\\0 & 0 & 4 & 4\\0 & 0 & 0 & -2\end{bmatrix}} \xrightarrow{R_1\leftrightarrow R_2} \underset{D}{\begin{bmatrix}4 & 0 & 5 & 2\\0 & 1 & 2 & 2\\0 & 0 & 4 & 4\\0 & 0 & 0 & -2\end{bmatrix}}$$

$\boxed{|D| = -32}$

(xii.xiie^xii) (STARE WELL) Let $A = \begin{bmatrix}1 & a & b & 2\\3 & c & d & 5\\2 & f & m & 7\\4 & h & w & 9\end{bmatrix}$. Given $|A| = 2019$. Then the solution set to the system

$|A| \neq 0$ unique solution

$$\begin{bmatrix}1 & a & b & 2\\3 & c & d & 5\\2 & f & m & 7\\4 & h & w & 9\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}2+5a+3b\\6+5c+3d\\4+5f+3m\\8+5h+3w\end{bmatrix}$$ is (unique solution)

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & C \end{array}$$
$$\left[\begin{array}{cccc|c}1 & a & b & 2 & 2+5a+3b\\3 & c & d & 5 & 6+5c+3d\\2 & f & m & 7 & 4+5f+3m\\4 & h & w & 9 & 8+5h+3w\end{array}\right]$$

Faculty information

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